

$s = ut$
 $2l = C_x \cdot t$
 $\frac{1}{2} = \frac{C_x}{2l}$
 \therefore Collision frequency $= \frac{C_x}{2l}$

\therefore Number of Collision per second $= \frac{C_x}{2l}$ — (2)

\therefore Change in momentum along x direction per second due to collisions $= 2m C_x \times \frac{C_x}{2l} = \frac{m C_x^2}{l}$ — (3)

\therefore Change in momentum per second due to the collision of one gas molecule on two opposite faces along x axis $= \frac{m C_x^2}{l} + \frac{m C_x^2}{l} = \frac{2m C_x^2}{l}$

Therefore the rate of change in momentum due to the collision per molecule on six faces of the cube $= \frac{2m C_x^2}{l} + \frac{2m C_y^2}{l} + \frac{2m C_z^2}{l}$
 $= \frac{2m}{l} (C_x^2 + C_y^2 + C_z^2)$

Therefore the rate of change in momentum for n molecules $= \frac{2mn C^2}{l}$ — (4)

According to Newton's 2nd law, the rate of change of momentum is equal to force.

And we also know that force per unit area is the Pressure of gas.

Hence, force $= \frac{2mn C^2}{l}$
 and area $= l^2$

\therefore Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{2mn C^2}{l \times l^2}$
 $= \frac{2mn C^2}{3 l^3}$ [$\because l^3 = V$]

\therefore Pressure $P = \frac{1}{3} \frac{m n C^2}{l^3} = \frac{1}{3} \frac{m n C^2}{V}$ — (5)

$\propto P = \frac{1}{3} \frac{M C^2}{V}$ [$\because m n = M$]

or $P V = \frac{1}{3} M C^2$ — (6)

The above equation is called the Kinetic gas equation.

or, $P = \frac{1}{3} \frac{M}{V} C^2$
 $= \frac{1}{3} \rho C^2$

Where ρ is density ($\rho = \frac{M}{V}$)

Relation between Kinetic Energy and Temperature of a gas.

The relationship between Kinetic Energy and Temperature can be derived from the Kinetic gas equation.

We know from Kinetic gas equation

$$PV = \frac{1}{3} mnc^2 \quad \text{--- (1)}$$

$$\text{or } PV = \frac{2}{3} \times \frac{1}{2} mnc^2$$

$$\text{or, } PV = \frac{2}{3} \times \frac{1}{2} Mc^2 \quad \text{--- (2) } \left[\begin{array}{l} \because mn = M \\ M = \text{Mass of the gas} \end{array} \right]$$

$$\therefore PV = RT \quad (\text{For 1 mole of gas}) \quad \text{--- (3)}$$

From eqn (1) and (3)

$$RT = \frac{2}{3} \times \frac{1}{2} Mc^2$$

$$RT = \frac{2}{3} \times K.E.$$

$$\therefore K.E = \frac{3}{2} RT$$

$$\left[\because K.E = \frac{1}{2} Mc^2 \right]$$

where m is Mass
& c is Velocity

$$\text{or } K.E \propto T \quad \left[\text{where } \frac{3}{2} R \text{ is Constant} \right]$$

So, It is clear that K.E of translation of an ideal gas is independent of the nature of the gas and its pressure. It depends only upon the temperature of the gas.

Derivation of the gas-law's on the basis of Kinetic gas equation.

(1) Derivation of the Boyle's Law: -

According to Kinetic theory of gases, the average Kinetic energy ($\frac{1}{2} mnc^2$) is directly proportional to absolute Temperature (T)

$$\text{i.e. } \frac{1}{2} mnc^2 = RT$$

$$\frac{3}{2} \times \frac{1}{2} mnc^2 = RT$$

$$\frac{3}{2} P.V = RT$$

$$\text{or } PV = \frac{2}{3} RT$$

$$\left[\because \frac{1}{2} mnc^2 = PV \right]$$

Therefore, the product of Pressure and Volume is a Constant at a Constant temperature.